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NOTE

A Note on List Arboricity

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A celebrated recent theorem of Galvin [2] asserts that for any bipartite graph, its chromatic index and its list-chromatic index are equal. This can be formulated as a property of the "independence system" of matchings in a bipartite graph. In private communication, Stefan Burr asked which other independence systems had this property, and in particular whether the independence system of forests in a graph had the property. We shall show that the latter is true. More precisely, we prove the following.

THEOREM 1. *Let G be a graph whose edges can be coloured with k colours so that no circuit of G is monochromatic. For each edge e let $P(e)$ be a list of k colours. Then each edge e can be coloured from its list $P(e)$ so that no circuit of G is monochromatic.*

This follows immediately from the following matroid theorem applied to the polygon matroid of G.

THEOREM 2. *Let M be a matroid so that $E(M)$ can be partitioned into k independent sets. For each $e \in E(M)$ let $P(e) \subseteq \{1, 2, ...\}$ with $|P(e)| \geqslant k$. Then $E(M)$ can be partitioned into independent sets X_i $(i = 1, 2, ...)$ so that $i \in P(e)$ for all $i \geqslant 1$ and all $e \in X_i$.*

Proof. We may assume that $P(e) \subseteq \{1, ..., n\}$ for all $e \in E(M)$. For $1 \leqslant i \leqslant n$, let $Q_i = \{e \in E(M): i \in P(e)\}$. Let M_i be the restriction of M to Q_i

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and let rk_i be the rank function of M_i. Let $F_1, ..., F_k$ be disjoint independent sets with union $E(M)$. Then, for any $X \subseteq E(M)$,

$$|X| = \sum_{1 \leqslant j \leqslant k} |F_j \cap X| \leqslant \sum_{1 \leqslant j \leqslant k} \frac{1}{k} \sum_{1 \leqslant i \leqslant n} |F_j \cap X \cap Q_i|$$

since every element belongs to Q_i for at least k values of i. But

$$|F_j \cap X \cap Q_i| \leqslant rk_i(X)$$

and so

$$|X| \leqslant \sum_{1 \leqslant i \leqslant n} \frac{1}{k} \sum_{1 \leqslant j \leqslant k} rk_i(X) = \sum_{1 \leqslant i \leqslant n} rk_i(X).$$

The result follows from the "matroid union" theorem [1,3].

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